

Creating And Solving Model Of Linear Equation Through The Balance At Junior Secondary Class

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Abstract

Algebra is one of the most difficult subject felt by most students and this strand is just started given to the students at early junior secondary school. Infact, if we can manage the algebra subject wisely and attractively, it can be predicted that the students' conceptual understanding algebra would be relatively improved.

A study was conducted to the Year 7 students at a Junior Secondary of Laboratory School of UPI. The class was set to learn the linear equation topic by using balance (scales).

Through a weighing activity, the students were able to design linear equation models. They followed a guidelines created by the teacher and researcher. The students were not only able to create linear equation models, but also were able to solve mathematical model of linear equation. By using steps of balance (weighing), finally the students were able to know the weight of a hidden quantity.

A number of teachers were involved in an observation activity which were designed in a lesson study context. Information related to the students' reaction as well as the teachers' reaction toward the implementation of creating and designing models of linear equation. The information were analysed qualitatively. The results indicate that introducing the linear equation through the scale (balance) were responded positively by the students. A brief interview with the students indicated that the students fluently could solve linear equation, and find the value of variable which infact as a weight variable. This variable seemed to be the weight of hidden variable as the solution of the linear equation.

Moreover, the students were able to interpret the process of weighing to the form of linear equation, since then the students solved it and found the solution of the problem.

While other teachers as observers at the lesson gave comments that the model teacher had practiced the concept of linear equation by using unusual way of teaching.

Intuitively they solved the linear equation by using step by step of weighing process and determined how much weight of an object. The process of weighing and thinking are parallel to solving a linear equation.

Data of test results regarding the linear equation indicated that the students' understanding of linear equation improved. The researchers recommend to use the balance (scales) as an alternative to teach the topic of linear equation.

Keywords: Balance, realistic, and lesson study.

I. INTRODUCTION

This research project is part of study to develop the teaching materials based on the realistic mathematics perspective in Junior Secondary School in Bandung, West Java. The topic of teaching materials was a linear equation. The teaching strategies to be used and developed were using balance (scale) for introducing process of solving equations.

Realistic Mathematics Education is a broad term for a teaching and learning

theory which is based on using problems taken from day-to-day experiences rather than on abstract mathematics rules. RME incorporates views of what mathematics is, how students learn it, and how it should be taught. An important driving force is to facilitate the view of mathematics as ‘a human activity (Freudenthal, 1991) rather than as subject matter which should be transferred to learners (Freudenthal, 1968). Mathematics should not be presented as a ready-made product (de Lange, 1987; Becker & Selter, 1996), nor begin with the formal system of rules and procedures; rather, students should be encouraged to re-invent key ideas in mathematics for themselves (Clarke, Clarke & Sullivan, 1996, p. 1225).

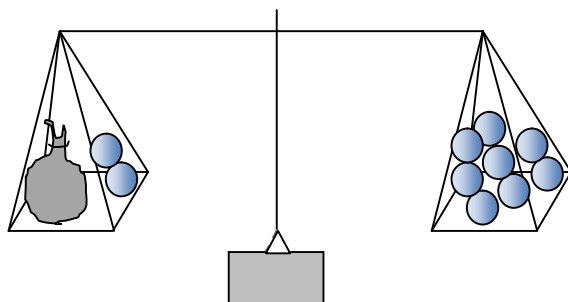
In conducting instruction of linear equation concept, a number of marbles were stored in a dark plastic bag for several bags. Some other marbles were used to make a scale to be balance. By using these bag of marbles, an unknown number of marbles in plastic bag to be considered as variable. Students have to find them out how many marble(s) inside the plastic bag. Weighing process is a process of how to find a number of marble(s) in a bag. Knowing how many marble(s) can be equated as solving equation process. By recognizing balancing process, students are cognitively helped to think a model of linear equation. They are also helped for step by step to solve mathematical problems. Model teacher and researcher work collaboratively to design the teaching materials as well as worksheets to implement the design in the lesson study format.

A number of teachers as members of MGMP of mathematics witnessed how the lesson take place in the real classroom situations. How were the student’s activities during tyhe lesson? And how was students solve linear equation by using scale (balance)? Students of SMP Laboratory School of UPI became subject for study. They were enthusiasm to use this ‘new’ approach in mathematics teaching. The students were skillful to translate the weighing process into solution process of equations.

They felt that they learn the lesson differently from the usual learning process. The members of matematics MGMP have positive impression toward this teaching approach in mathematics compare to the current and usual teaching approach that they use at schools.

Rather than observing and studying ‘what is’ happening currently in teaching and assessing of mathematics in the classroom, we would like to create ‘what ought to be’ happening. This is what Silver (1989) calls *transformative research*. This kind of research is studied in more detail by the community of realistic mathematics educators (Gravemeijer, 2000; De Lange, 2000; Freudenthal, 1991). Design of this teaching approach would be one of efforts to improve the quality of learning in the classroom.

Notice the following solution problems (Turmudi, 2010).



How do you know how many marble(s) inside a plastic bag? Intuitively students know how to find the number of marble(s) in the plastic bag. By giving an argument, that both sides above are equivalent, it means that a number of marble in the left hand are the same as a number of marble(s) at the right hand with an assumption that the weight of each marble is uniform. There was no weight (weighless) of plastic bag to store the marble in bag. In the right hand of scale, there are 8 marbles, in order to be balance, so in the left hand we should have eight marbles as well. The marble appears two of them, so we can conclude that in the bag we should have 6 marbles.

Mathematically this thinking process are equivalent to $x + 2 = 8$ and solve equation for x by taking them two marbles out from both sides. Therefore the situation was still balance, and $x = 6$.

Above situation indicated that the process of weighing was an initial process to understand the concept algebra, particularly in solving linear equations.

II. RESEARCH METHOD

Algebra has its historical roots in the study of general methods for solving equations (NCTM, 2000: 37). Drawing students' attention to the relationship between algebra and practical knowledge has traditionally been conducted through what we called as "algebra word problems". However, this approach has been criticized because of contrive nature of most problems which are presented to the students. There are a number ways to introduce and solve equations.

In order to be a meaningful and useful, lessons should use the context of daily life as a starting point to learn mathematics. But problems can also arise from mathematical situations that are meaningful and natural to the students. The word 'real' refers to the experientially real rather than to 'real world'; this is what RME theory calls 'contextual principle' (Drijvers, 2000), or didactical phenomenology (Freudenthal, 1983; 1991). Bakker (2000) defines didactical phenomenology as the study of the relationship between mathematical concepts and the phenomena in which they arise with respect to

process of teaching and learning (p.2).

This study paper is part of qualitative research to improve the teachers' aweranness to innovation of mathematics instruction in Bandung West Java. Data collected are consisted of data from planning, observation, reflection, and data of students' performance in mathematics. All data would be analysed qualitatively.

Initially the teachers of mathematics were designed to join a workshop-training in teaching and learning mathematics using modelling based on the realistic mathematics perspective. Experiences in designing teaching materials based on the realistic perspective are presented to the teachers. One of the experiences is designing a balance (scale) for weighing things for introducing linear equations. This approach is real, contextual, and hands-on activities for the students to develop their understanding about equation concepts. A number kinds of balance were introduced to the teachers include digital balance, scale, and analog.



(a)



(b)



(c)



(d)

However, balance (a) or (b) is the most appropriate balance for introducing linear equation concepts.

A member of lesson study program interested to design the teaching materials for introducing algebra expression as well as linear equation using balance (scale). This member later on is called as model teacher. The model teacher made balance model using a platic plate, string, marbles, plastic bag, and cloth hangers. When design is presented by the teacher in a simulation of workshop as part of lesson study program,

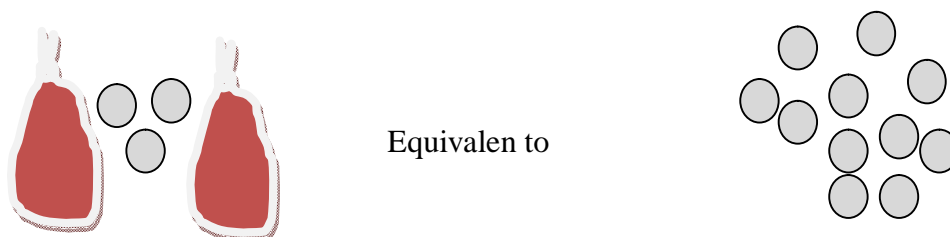
the design of teaching materials indicated that design is representative for introducing how the students recognize and understand the concepts of linear equations. Each step of ballancing activity is isomorphic to solving process of linear equations. Even the terms *add to* or *subtract from* two sides of an equation with the same numbers, according to the proponent of algebra called *moqabala* (van Den Waerden, 1985) similarize to performing algebraic operations, in the reality we can observe in ballancing (weighing) processes.

Learning process syntax consisted of opening part, learning implementation, result presentation, and conclusion. In the opening session, the teachers introduced learning design that was uncommon, different from the daily learning activities. Junior

secondary students involved in this research are year 8th students of Laboratory School of UPI. The class consisted of 22 students. The students were grouped into 7 groups of 3 or 4 students. Each group got a set of teaching materials which consisted of teacher-made balance model using a plastic plate, string, marbles, plastic bag, and cloth hangers. A worksheet was also provided for each group that have to be discussed in group.

The students are listening to the teacher's explanation with regard to what should students do with ballancing and weighing activities, and how should students answer and fill out the worksheets.

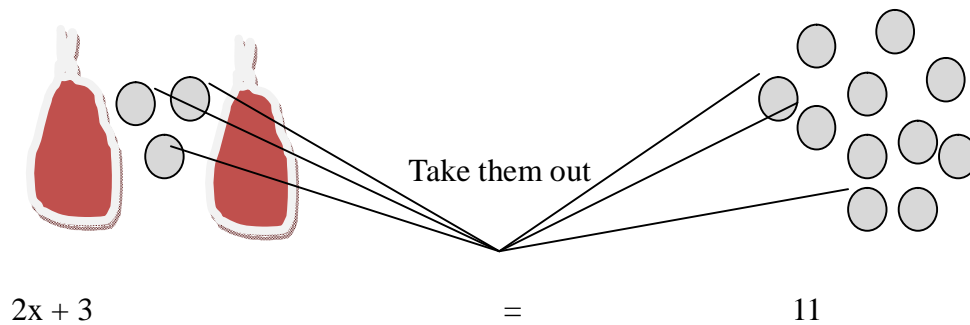
For example, the teacher presents the simple equation $2x + 3 = 11$, by using balance, the students compare this situation as illustrated in the following figure.



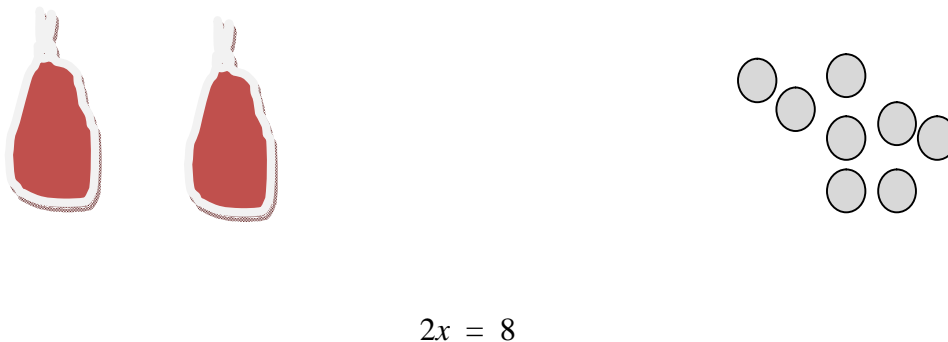
In order to know how many marble(s) inside the bag, of course the students have to know first that both sides have the same number of marbles, with the assumption that all marbles have the same weight, and the bag of plastic have no weight (or can be omitted).

In the learning session part, the students try to understand the concept of linear equation by using weighing process. Step by step the students in a group simplify equation by considering weighing the marbles.

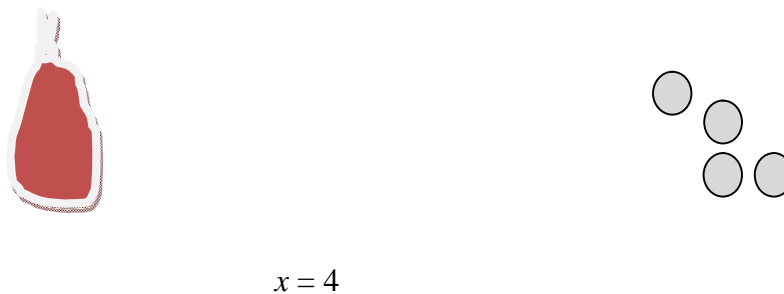
Step 1:



Step 2:



Step 3



Algebraic thinking that be helped by using balance (scale) enable students to relate algebraic thinking process and cognitive thinking process experiencing by the students using weighing. Qualitative data of weighing (ballancing) process conducted by the students, student's understanding of mathematical concepts, students' interaction among them, and teachers reaction toward this new teaching approach were collected and analysed qualitatively.

Conclusion of qualitative data indicate that students along with the teachers (spectators of the lesson) expected a learning (and teaching) model that activate the learners to contruct their own knowledge of mathematical concepts.

III.RESULT

As previously mentioned that this research is process of creating and solving linear equation through weighing (scale) activities. This study is part of qualitative research in the context of lesson study. Results of the study can be clasified in three parts: How was the teachers (MGMP members) response toward this “new” teaching approach in teaching linear equation? How was students reaction toward this teaching approach? And how was the students’ achievement in linear equation concepts? Data collected either from the students or from the teachers have been used to answer those question above. Research data consisted of interview data with the students and the teachers (member of MGMP), observation data, fieldnote in reflection session, and test result.

Interview data with the teachers indicated that LS participants enthusiastically involved in this teaching. The teachers of MGMP members role as observer have eager to help students to give scaffolding. However, they could not help them, as they position only as purely observer, so that the observer only note what students did and thought with regard to learn linear equation by using scale.

Model teacher always monitored how were the students learn, always give guide when they stuck with their learning by using scale (weighing) process which was unusual for them.

Based on the observation results, almost all the students in their group involved in contributing their thinking to solve tasks given to the group. Stdents in a group worked collaboratively to discuss the results of their work to find out the solution of the linear equation.

Initially they worked as previously mentioned, by using weighing process. Step by step procedure for balancing they done which finally they solved the problem by recognizing how many marble(s) inside the plastic bag. This situation is parallel with the solution of equation to find the value of variable. Therefore the students can take into account both process the weighing process and solution process of linear equation. For two groups of students that has barriers in discussion process, model teacher gave help (scaffolding) to make more fluence to solve problems given to them.

For about 40% time of the lesson was used by the students for discussion and working collaboratively to weigh the marble(s), to read worksheet, to manipulate the materials, to discuss the results, to predic the result, to make decision, to debate among

them, to make a generalization, and to make a conclusion.

For about 30% of time was used to present the result of discussion in front of the classroom. Other group gave comments to the results of presentation. Ideally all the group representative have to present the results of discussion. In this case model teacher only choose one group representative that have the same category. It means that the teacher categorized the students respons into three different categories.

In the reflection session, a number of teachers commented to this ‘new’ teaching approach. Almost all visitors appreciated to the model teachers that can perform well in teaching algebra. Usually this topic is taught by majority of teachers using direct teaching, the teacher give an example how to solve the problems, then give excercise for the students, and then the test. In this teaching approach, the model teacher use indirect teaching, the students worked themselves in a group, reflected their understanding, and their thinking about linear equation. Measuring how much knowledge that students know about linear equations.

Some teachers commented that the formal part of students thinking in equation was less frequently discussed by model teacher. Students worked more frequently in informal mathematics. Hence the teachers worried that the students cannot perform algebraic operations, if they work only using scale (balance). Though the students were skillful to operate and to weigh the marble(s) in order to know the number of marble inside the plastic bag, the students were still less competence to solve mathematical problems. Therefore, it would be suggested that the model teacher have to provide long enough time for the students to excercise in manipulating and solving linear equation problems.

When we asked them “Do you like learning mathematics (specifically in equation) using scale (weghing process)? Most of them response “Yes I like this new approach’ because we can directly process and relate the daily activities with mathematical situations. Mathematical thinking process was naturally experiencing by the learners.

Data about students’ understanding of linear equation were obtained from the model teacher that implement the instructional design.

Generally students know better about linear equation that was designed collaboratively among researcher and model teacher.

However, it would be better if we give students opportunity to assimilate and accomodate their understanding of linear equation, and to compare between weighing marble(s) process and solving linear equation.

IV. DISCUSSION

Results of this study indicated that either students or teachers paid more positive attention to this 'new' teaching approach. Learning linear equation by using scale (balance) and weighing process has motivated the students to understand concept of variable. Number of marble(s) inside the plastic bag can be considered as variable concept.

When a variable is given the certain value, then an abstract properties of variable become more conreet for the students. The same as number of marbles in a plastic bag, by using weighing process and using equivalent process then the number of marbles in the bag can be obtained. In ordet to see that the results of weighing process is correct then the plastic bag can be opened, then it is the knowing process of variable.

Variable is usually known as unkown number. In an equation unknown number can be known by solving the equation. Understanding that the process of weighing is the same as process of *al jabr wa muqobala* from Alkhawarizm (van de Waerden, 1985), the step process knowing variable was equivalent to solve an equation.

The philosophy underpinning the current curriculum influences the ways in which teachers teach. With regard to the effectiveness of their practice, the National Research Council (1989, cited by NCTM, 1991) summarized:

Effective teachers are those who can stimulate students to learn mathematics. Educational research offers compelling evidence that students learn mathematics well when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum; 'examine', 'represent', 'transform', 'solve', 'apply', 'prove', 'communicate'. This happens most readily when students work in groups, engage in discussion, make a presentation, and in other way take charge of their own learning. (p.2)

One thing to distinguish weighing process from solution an equation is that in weighing process we work with the real things. The number n this situation is not negative. Whereas in the weighing (scalling) was not working with negative numbers. This is the role of context (weighing) that can be used for "bridging" to understand the

abstract concept in algebra. For the negative number in algebra we did not recommend to use balance (weighing) for solving problem of linear equation. This situation is relevant to the statement of NCTM (2000) that emphasize on the high order thinking in mathematics. Furthermore NCTM suggests that in the middle grades students should work more frequently with algebraic symbols than in lower grades, the students need to use a concrete bridging for them to think. In this situation students should develop an initial understanding of several different meanings and uses of variables through representing and quantities in a variety of problem situations. In the middle grades (or SMP levels) students should learn to recognize and generate equivalent expressions, solve linear equations, and use simple formulas. According to Schoenfeld and Archavi (1988) that students' understanding of variable should go far beyond recognizing letters can be used to stand for unknown numbers in equations.

Classroom situation in this research is more interactive than in usual situation. There are two steps to be done by the students in the cooperative group setting. Initially they have to know what should be done with worksheet, they have to make a decision in a group what should they do for solving the problems, then they have to weigh perhaps to put down from, or put up the marble on, both sides of scale in order to be balance.

Students' understanding of the linear equation is not just skillful to weigh things, and to know thing inside the bag, but students are expected to be able to manipulate and perform algebraic operations. Students are expected to be able to solve mathematical equation (linear equation) not only by sing scale (balance) but also by using algebraic operation in more formal mathematical expressions.

V. CONCLUSION AND SUGGESTION

Based on the description above and discussion section the research results indicate:

- (1) Students enthusiastically learn mathematics (linear equation) by using balance (scale).
- (2) Students felt that this new approach was different from usual teaching that they used before.
- (3) Result of this teaching approach motivated the students to always relate mathematical concepts with observation and exploration data.
- (4) The teachers as members of MGMP felt that this example of teaching models inspired them to find other examples of teaching model.

- (5) In implementing this ‘new’ teaching approach, it is hoped that this new approach is not the first and the last. Teacher and researcher always get in touch for designing and trying out the teaching mathematics, so the teachers have a lot of examples in applying new teaching approaches.

Based on the conclusions, it may be suggested for the teachers to learn more teaching approaches that enable students to construct their own knowledge. The teaching approach that involved students in hands-on activities such as weighing process using scale (balance). Innovation in mathematics instruction are expected by most teachers, therefore various strategies and contexts for learning mathematics need to be trained for mathematics teachers to enable them teach by using new paradigm.

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